

New Model of $\mathcal{N}=8$ Superconformal Mechanics

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Abstract

Using an $\mathcal{N}=4, d=1$ superfield approach, we construct an $\mathcal{N}=8$ supersymmetric action of the self-interacting off-shell $\mathcal{N}=8$ multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$. This action is found to be invariant under the exceptional $\mathcal{N}=8, d=1$ superconformal group $F(4)$ with the R -symmetry subgroup $SO(7)$. The general $\mathcal{N}=8$ supersymmetric $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ action is a sum of the superconformal action and the previously known free bilinear action. We show that the general action is also superconformal, but with respect to redefined superfield transformation laws. The scalar potential can be generated by two Fayet-Iliopoulos $\mathcal{N}=4$ superfield terms which preserve $\mathcal{N}=8$ supersymmetry but break the superconformal and $SO(7)$ symmetries.

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1 Introduction

Superconformal mechanics (SCM) plays an important role in a wide circle of intertwining problems related to black holes, $\text{AdS}_2/\text{CFT}_1$, super Calogero-Moser models, branes, etc [1]–[14]. While the $\mathcal{N}=2$ and $\mathcal{N}=4$ SCM models were constructed long ago [1]–[10], much less is known about the higher $\mathcal{N}>4$ cases, in particular, the $\mathcal{N}=8$ one. This is related to the fact that the number of admissible non-equivalent $d=1$ superconformal groups is growing with \mathcal{N} . For instance, there is only one choice for the $\mathcal{N}=2$ superconformal group, $SU(1,1|1) \sim OSp(2|2)$, whereas there are three possibilities, $D(2,1;\alpha)$, $OSp(4^*|2)$ and $SU(1,1|2)$, in the $\mathcal{N}=4$ case¹ and four in the $\mathcal{N}=8$ case: $OSp(8|2)$, $OSp(4^*|4)$, $F(4)$ and $SU(1,1|4)$ [15, 16].

The $\mathcal{N}=8$ superconformal actions of the off-shell multiplets $(\mathbf{5}, \mathbf{8}, \mathbf{3})$ and $(\mathbf{3}, \mathbf{8}, \mathbf{5})$ were explicitly given in [14] in terms of the properly constrained $\mathcal{N}=4$, $d=1$ superfields. It was found that in both cases the underlying superconformal symmetry is $OSp(4^*|4)$. It is interesting to construct superconformal models associated with other $\mathcal{N}=8$, $d=1$ superconformal groups. An example of such a system is presented here. It is the $\mathcal{N}=8$ supersymmetric mechanics model associated with the off-shell multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ from the list of [17]. The underlying $\mathcal{N}=8$ superconformal symmetry is the exceptional supergroup $F(4)$ with the R -symmetry subgroup $SO(7)$. In the $\mathcal{N}=4$ superfield approach which we use throughout the paper, the “manifest” superconformal group is $D(2,1;-1/3) \subset F(4)$.

In terms of $\mathcal{N}=4$ superfields, the multiplet in question amounts to a sum

$$(\mathbf{1}, \mathbf{8}, \mathbf{7}) = (\mathbf{1}, \mathbf{4}, \mathbf{3}) \oplus (\mathbf{0}, \mathbf{4}, \mathbf{4}). \quad (1.1)$$

While for the multiplet $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ one can write manifestly $\mathcal{N}=4$ supersymmetric actions in the ordinary $\mathcal{N}=4$ superspace [3, 18], actions of the fermionic $\mathcal{N}=4$ multiplet $(\mathbf{0}, \mathbf{4}, \mathbf{4})$ are naturally written in the analytic harmonic $\mathcal{N}=4$ superspace [19, 14, 17]. In order to construct the $\mathcal{N}=8$ supersymmetric actions of the multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ we use the $\mathcal{N}=4$, $d=1$ harmonic superspace description for both $\mathcal{N}=4$ multiplets in (1.1). A sum of the free $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ action [17] and the superconformal action constructed here yields the most general $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ action. We present the relevant component off-shell action and show that it agrees with that found in [20] by a different method. Surprisingly, the general action is also superconformal, though with respect to redefined superfield transformation laws. For both $\mathcal{N}=4$ multiplets one can construct $\mathcal{N}=8$ supersymmetric Fayet-Iliopoulos terms which, however, break superconformal symmetry.

2 Preliminaries: $\mathcal{N}=4$, $d=1$ harmonic superspace

The harmonic analytic $\mathcal{N}=4$ superspace [21, 19, 22, 23] is parametrized by the coordinates

$$(\zeta, u) = (t_A, \theta^+, \bar{\theta}^+, u_i^\pm), \quad u^{+i}u_i^- = 1. \quad (2.2)$$

They are related to the standard $\mathcal{N}=4$ superspace (central basis) coordinates $z = (t, \theta_i, \bar{\theta}^i)$ as

$$t_A = t - i(\theta^+\bar{\theta}^- + \theta^-\bar{\theta}^+), \quad \theta^\pm = \theta^i u_i^\pm, \quad \bar{\theta}^\pm = \bar{\theta}^i u_i^\pm. \quad (2.3)$$

The $\mathcal{N}=4$ covariant spinor derivatives and their harmonic projections are defined by

$$D^i = \frac{\partial}{\partial \theta_i} + i\bar{\theta}^i \partial_t, \quad \bar{D}_i = \frac{\partial}{\partial \bar{\theta}^i} + i\theta_i \partial_t, \quad \overline{(D^i)} = -\bar{D}_i, \quad \{D^i, \bar{D}_k\} = 2i\delta_k^i \partial_t, \quad (2.4)$$

$$D^\pm = u_i^\pm D^i, \quad \bar{D}^\pm = u_i^\pm \bar{D}^i, \quad \{D^+, \bar{D}^-\} = -\{D^-, \bar{D}^+\} = 2i\partial_{t_A}. \quad (2.5)$$

¹Actually, the supergroups $OSp(4^*|2)$ and $SU(1,1|2)$ can be treated as special cases of $D(2,1;\alpha)$.

In the analytic basis, the derivatives D^+ and \bar{D}^+ are short,

$$D^+ = \frac{\partial}{\partial \theta^-}, \quad \bar{D}^+ = -\frac{\partial}{\partial \bar{\theta}^-}. \quad (2.6)$$

The analyticity-preserving harmonic derivative D^{++} and its conjugate D^{--} are given by

$$\begin{aligned} D^{++} &= \partial^{++} - 2i\theta^+\bar{\theta}^+\partial_{t_A} + \theta^+\frac{\partial}{\partial \theta^-} + \bar{\theta}^+\frac{\partial}{\partial \bar{\theta}^-}, \\ D^{--} &= \partial^{--} - 2i\theta^-\bar{\theta}^-\partial_{t_A} + \theta^-\frac{\partial}{\partial \theta^+} + \bar{\theta}^-\frac{\partial}{\partial \bar{\theta}^+}, \quad \partial^{\pm\pm} = u_i^\pm \frac{\partial}{\partial u_i^\mp}, \end{aligned} \quad (2.7)$$

and become the pure partial derivatives $\partial^{\pm\pm}$ in the central basis. They satisfy the relations

$$[D^{++}, D^{--}] = D^0, \quad [D^0, D^{\pm\pm}] = \pm 2D^{\pm\pm}, \quad (2.8)$$

where D^0 is the operator counting external harmonic $U(1)$ charges. The integration measures in the full harmonic superspace (HSS) and its analytic subspace are defined as

$$\begin{aligned} \mu_H &= dudtd^4\theta = dudt_A(D^-\bar{D}^-)(D^+\bar{D}^+) = \mu_A^{(-2)}(D^+\bar{D}^+), \\ \mu_A^{(-2)} &= dud\zeta^{(-2)} = dudt_A d\theta^+ d\bar{\theta}^+ = dudt_A(D^-\bar{D}^-). \end{aligned} \quad (2.9)$$

3 The multiplets (1, 4, 3) and (0, 4, 4)

3.1 (1, 4, 3)

The off-shell multiplet **(1, 4, 3)** is described by a real $\mathcal{N}=4$ superfield $v(z)$ obeying the constraints [3]

$$D^i D_i v = \bar{D}_i \bar{D}^i v = 0, \quad [D^i, \bar{D}_i]v = 0. \quad (3.1)$$

The same constraints in HSS read [17]

$$D^{++}v = 0, \quad D^+ D^- v = \bar{D}^+ \bar{D}^- v = 0, \quad (D^+ \bar{D}^- + \bar{D}^+ D^-)v = 0. \quad (3.2)$$

The extra harmonic constraint guarantees the harmonic independence of v in the central basis.

Recently, it was shown [23] that this multiplet can be also described in terms of the real analytic gauge superfield $\mathcal{V}(\zeta, u)$ subjected to the abelian gauge transformation

$$\mathcal{V} \Rightarrow \mathcal{V}' = \mathcal{V} + D^{++}\Lambda^{--}, \quad \Lambda^{--} = \Lambda^{--}(\zeta, u). \quad (3.3)$$

In the Wess-Zumino gauge just the irreducible **(1, 4, 3)** content remains

$$\mathcal{V}_{WZ}(\zeta, u) = x(t_A) + \theta^+ \omega^i(t_A) u_i^- + \bar{\theta}^+ \bar{\omega}^i(t_A) u_i^- + 3i\theta^+ \bar{\theta}^+ A^{(ik)}(t_A) u_i^- u_k^-. \quad (3.4)$$

No residual gauge freedom is left. The original superfield $v(z)$ is related to $\mathcal{V}(\zeta, u)$ by

$$v(t, \theta^i, \bar{\theta}_k) = \int du \mathcal{V}\left(t - 2i\theta^i \bar{\theta}^k u_{(i}^+ u_{k)}^-, \theta^i u_i^+, \bar{\theta}^k u_k^+, u_l^\pm\right). \quad (3.5)$$

The constraints (3.1) are recovered as a consequence of the harmonic analyticity of \mathcal{V}

$$D^+ \mathcal{V} = \bar{D}^+ \mathcal{V} = 0. \quad (3.6)$$

We shall need a “bridge” representation of \mathcal{V} through the superfields $v(z)$ and $V^{--}(z, u)$

$$\mathcal{V} = v + D^{++}V^{--}, \quad v' = v, \quad V^{--'} = V^{--} + \Lambda^{--}. \quad (3.7)$$

The term $v(z)$ is just given by the expression (3.5). The analyticity conditions (3.6) imply

$$D^-v + D^+V^{--} = \bar{D}^-v + \bar{D}^+V^{--} = 0. \quad (3.8)$$

Below are some useful corollaries of (3.8), (3.6) and (2.5)

$$(D^+\bar{D}^- - \bar{D}^+D^-)v = -2D^+\bar{D}^+V^{--}, \quad (3.9)$$

$$D^+\bar{D}^+V^{--} = iD^{++}\left(\dot{V}^{--} + \frac{i}{2}D^-\bar{D}^-v\right) - i\dot{\mathcal{V}}, \quad (3.10)$$

$$D^+\left(\dot{V}^{--} + \frac{i}{2}D^-\bar{D}^-v\right) = \bar{D}^+\left(\dot{V}^{--} + \frac{i}{2}D^-\bar{D}^-v\right) = 0. \quad (3.11)$$

The general invariant action of the multiplet $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ reads

$$S_{gen}^{(v)} = \int dt d^4\theta \mathcal{L}_{gen}(v). \quad (3.12)$$

The free action corresponds to the quadratic Lagrangian

$$S_{free}^{(v)} = - \int dt d^4\theta v^2, \quad (3.13)$$

while the action invariant under the most general $\mathcal{N}=4, d=1$ superconformal group $D(2, 1; \alpha)$ (except for the special values of $\alpha=0, -1$) is [9]

$$S_{sc}^{(v)} = - \int dt d^4\theta (v)^{-\frac{1}{\alpha}}, \quad (3.14)$$

where, for the correct $d=1$ field theory interpretation, one must assume that v develops a non-zero background value, $v = 1 + \dots$. The transformation properties of some relevant objects under the conformal $\mathcal{N}=4$ supersymmetry $\subset D(2, 1; \alpha)$ are as follows [19, 23]²

$$\delta_{sc} D^{++} = -\Lambda_{sc}^{++} D^0, \quad \delta_{sc} D^0 = 0, \quad (3.15)$$

$$\delta_{sc} \mu_A^{(-2)} = 0, \quad \delta_{sc} \mu_H = \mu_H \left(2\Lambda_{sc} - \frac{1+\alpha}{\alpha} \Lambda_0 \right),$$

$$\delta_{sc} (dt d^4\theta) = -\frac{1}{\alpha} (dt d^4\theta) \Lambda_0, \quad \delta_{sc} du = du D^{--} \Lambda_{sc}^{++} \quad (3.16)$$

$$\delta_{sc} v = -\Lambda_0 v, \quad \delta_{sc} \mathcal{V} = -2\Lambda_{sc} \mathcal{V}, \quad (3.17)$$

where

$$\Lambda_{sc}^{++} = 2i\alpha(\bar{\varepsilon}^+\theta^+ - \varepsilon^+\bar{\theta}^+) \equiv D^{++}\Lambda_{sc}, \quad \Lambda_{sc} = 2i\alpha(\bar{\varepsilon}^-\theta^+ - \varepsilon^-\bar{\theta}^+), \quad (D^{++})^2\Lambda_{sc} = 0, \quad (3.18)$$

$$\Lambda_0 = (2\Lambda_{sc} - D^{--}\Lambda_{sc}^{++}) = 2i\alpha(\theta_i\bar{\varepsilon}^i + \bar{\theta}^i\varepsilon_i), \quad D^{++}\Lambda_0 = 0, \quad (3.19)$$

²Invariance under these transformations is sufficient to check $D(2, 1; \alpha)$ invariance since the rest of the $D(2, 1; \alpha)$ transformations is contained in the closure of the conformal and manifest Poincaré $\mathcal{N}=4, d=1$ supersymmetries.

and $\varepsilon^\pm = \varepsilon^i u_i^\pm$, $\bar{\varepsilon}^\pm = \bar{\varepsilon}^i u_i^\pm$, $\varepsilon^i, \bar{\varepsilon}_i$ being mutually conjugated Grassmann transformation parameters. Using (3.16), (3.17) and (3.18), (3.19), it is easy to check the $D(2, 1; \alpha)$ invariance of the action (3.14) and the covariance of the relation (3.5).

One can also construct an $\mathcal{N}=4$ supersymmetric Fayet-Iliopoulos (FI) term

$$S_{FI}^{(v)} = i \int dud\zeta^{(-2)} c^{+2} \mathcal{V} \quad c^{+2} = c^{ik} u_i^+ u_k^+, \quad [c] = cm^{-1}, \quad (3.20)$$

which produces a scalar potential after elimination of the auxiliary field $A^{(ik)}$ in the sum of (3.12) and (3.20). This term is superconformal only for the special choice $\alpha=0$ [23].

3.2 (0, 4, 4)

The multiplet $(\mathbf{0}, \mathbf{4}, \mathbf{4})$ comprises 4 fermionic fields and 4 bosonic auxiliary fields. It is described off shell by the fermionic analytic superfield Ψ^{+A} , $(\widetilde{\Psi^{+A}}) = \Psi_A^+$, obeying the constraint [19]:

$$D^{++}\Psi^{+A} = 0 \Rightarrow \Psi^{+A} = \psi^{iA} u_i^+ + \theta^+ a^A + \bar{\theta}^+ \bar{a}^A + 2i\theta^+ \bar{\theta}^+ \dot{\psi}^{iA} u_i^-. \quad (3.21)$$

With respect to the doublet index A ($A = 1, 2$), it is transformed by some extra (“Pauli-Gürsey”) group $SU(2)_{PG}$ which commutes with $\mathcal{N}=4$ supersymmetry. The requirement of superconformal covariance of the constraint (3.21) uniquely fixes the superconformal $D(2, 1; \alpha)$ transformation rule of Ψ^{+A} , for any α , as

$$\delta_{sc} \Psi^{+A} = \Lambda_{sc} \Psi^{+A}. \quad (3.22)$$

In the central basis, the constraint (3.21) implies

$$\Psi^{+A}(z, u) = \Psi^{iA}(z) u_i^+, \quad (3.23)$$

and the analyticity conditions $D^+ \Psi^{+A} = \bar{D}^+ \Psi^{+A} = 0$ amount to

$$D^{(i} \Psi^{k)A}(z) = \bar{D}^{(i} \Psi^{k)A}(z) = 0. \quad (3.24)$$

The free action of Ψ^{+A} ,

$$S_{free}^{(\psi)} = \int dud\zeta^{(-2)} \Psi^{+A} \Psi_A^+, \quad (3.25)$$

is not invariant under $D(2, 1; \alpha)$ (except for the special case of $\alpha=0$). However, we can construct a superconformal invariant by coupling Ψ^{+A} to the $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ multiplet [23]

$$S_{(sc)}^{(\psi)} = \int dud\zeta^{(-2)} \mathcal{V} \Psi^{+A} \Psi_A^+. \quad (3.26)$$

This action is superconformal at any α , and it also respects the gauge invariance (3.3) as a consequence of the constraint (3.21). Assuming that $\mathcal{V} = 1 + \tilde{\mathcal{V}}$, $v = 1 + \tilde{v}$, (3.26) can be treated as a superconformal generalization of the free action (3.25). A simple analysis based on dimensionality and on the Grassmann character of the superfields Ψ^{+A} , $\Psi^{-A} = D^{--} \Psi^{+A}$ shows that no self-interaction of the multiplet $(\mathbf{0}, \mathbf{4}, \mathbf{4})$ can be constructed. Also, the coupling (3.26) is the only possible coupling of this fermionic multiplet to the multiplet $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ preserving the canonical number of fields with time derivative in the component action (no more than two for bosons and no more than one for fermions).

The only additional $\mathcal{N}=4$ invariant one can construct is the appropriate FI-type term

$$S_{FI}^{(\psi)} = \int dud\zeta^{(-2)} (\theta^+ \xi_A \Psi^{+A} + \bar{\theta}^+ \bar{\xi}^A \Psi_A^+), \quad (3.27)$$

$\xi_A, \bar{\xi}^A$ being $SU(2)_{PG}$ breaking constants. This term is superconformal at $\alpha = -1$ [23].

4 $\mathcal{N}=8$ supersymmetry

As shown in [17], one can define the hidden $\mathcal{N}=4$ supersymmetry³

$$\delta_\eta v = -\eta_{iA} \Psi^{iA}, \quad \delta_\eta \Psi^{iA} = \frac{1}{2} \eta_k^A (D^i \bar{D}^k - \bar{D}^i D^k) v. \quad (4.1)$$

It commutes with the explicit $\mathcal{N}=4$ supersymmetry and so forms, together with the latter, $\mathcal{N}=8$, $d=1$ Poincaré supersymmetry. It is easy to check the compatibility of (4.1) with the constraints (3.1), (3.24). The same transformations, being rewritten in HSS, read

$$\delta_\eta v = \eta^{-A} \Psi_A^+ - \eta^{+A} \Psi_A^-, \quad \delta_\eta \Psi^{+A} = \eta^{-A} D^+ \bar{D}^+ v - \frac{1}{2} \eta^{+A} (D^+ \bar{D}^- - \bar{D}^+ D^-) v, \quad (4.2)$$

where $\eta^{\pm A} = \eta^{iA} u_i^\pm$. The appropriate transformation of the analytic prepotential \mathcal{V} is

$$\delta_\eta \mathcal{V}(\zeta, u) = 2\eta^{-A} \Psi_A^+(\zeta, u). \quad (4.3)$$

As expected, (4.3) closes on the time derivative of \mathcal{V} only modulo a gauge transformation:

$$[\delta_\eta, \delta_{\eta'}] \mathcal{V} = 2i(\eta'_{iA} \eta^{iA}) (\dot{\mathcal{V}} - D^{++} \tilde{\Lambda}^{--}), \quad \tilde{\Lambda}^{--} = \dot{V}^{--} + \frac{i}{2} D^- \bar{D}^- v, \quad (4.4)$$

where we used the identities (3.9)-(3.11) and anticommutation relations (2.5).

Now we wish to construct the most general action of the multiplets $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ and $(\mathbf{0}, \mathbf{4}, \mathbf{4})$ which would enjoy the hidden supersymmetry (4.1), (4.2) and so present the $\mathcal{N}=4$ superfield form of the general $\mathcal{N}=8$ supersymmetric action of the multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$.

A convenient starting point of such a construction is offered by the Ψ -actions (3.25) and (3.26) in view of their uniqueness. The $\mathcal{N}=8$ completion of the free action (3.25) was found in [17]. The variation of (3.25) under (4.2) can be written as

$$\delta_\eta S_{free}^{(\psi)} = 4 \int \mu_A^{(-2)} D^+ \bar{D}^+ v (\eta^{-A} \Psi_A^+) = 4 \int \mu_H v (\eta^{-A} \Psi_A^+) = -2 \int dt d^4 \theta v (\eta^{iA} \Psi_{iA}), \quad (4.5)$$

where we used the relation (2.9), constraint (3.21) and the harmonic independence of v in the central basis. This variation is cancelled by that of the free v action (3.13), so the action

$$S_{free}^{(\mathcal{N}=8)} = \frac{1}{2} \left(\int \mu_A^{(-2)} \Psi^{+A} \Psi_A^+ - \int dt d^4 \theta v^2 \right) \quad (4.6)$$

is $\mathcal{N}=8$ supersymmetric. It breaks superconformal symmetry, since its first term is invariant under $D(2, 1; \alpha = -1/2)$ (see (3.14)), while the second one is invariant under $D(2, 1; \alpha=0)$.

Now let us promote the interaction action (3.26) to an $\mathcal{N}=8$ invariant. To calculate the variation $\delta_\eta S_{sc}^{(\psi)}$, we firstly note that it is fully specified by the variation $\delta_\eta \Psi^{+A}$, since $\delta_\eta \mathcal{V}$ does not contribute because of the nilpotency property $(\Psi^{+A})^3 = 0$. Then, using (3.9), we represent

$$\delta_\eta \Psi^{+A} = D^+ \bar{D}^+ (\eta^{-A} v + \eta^{+A} V^{--}), \quad (4.7)$$

³The relative sign between these two variations was chosen so as to have the same closure for the hidden supersymmetry as for the manifest one.

restore the full superspace integration measure in $\delta_\eta S_{sc}^{(\psi)}$ and rewrite this variation as

$$\begin{aligned} \delta_\eta S_{sc}^{(\psi)} &= 2 \int \mu_H \left[v^2 (\eta^{-A} \Psi_A^+) + v D^{++} V^{--} (\eta^{-A} \Psi_A^+) \right. \\ &\quad \left. + v V^{--} (\eta^{+A} \Psi_A^+) + V^{--} D^{++} V^{--} (\eta^{+A} \Psi_A^+) \right], \end{aligned} \quad (4.8)$$

where the bridge representation (3.7) for \mathcal{V} was used. Taking into account the harmonic constraint (3.21) and the properties $D^{++} \eta^{-A} = \eta^{+A}$ and $D^{++} \eta^{+A} = 0$, we observe that all terms in (4.8) except for the first one are reduced to a total harmonic derivative, whence

$$\delta_\eta S_{sc}^{(\psi)} = 2 \int \mu_H v^2 (\eta^{-A} \Psi_A^+) = - \int dt d^4 \theta (\eta^{iA} \Psi_{iA}) v^2. \quad (4.9)$$

This is cancelled out by the variation of $-1/3 \int dt d^4 \theta v^3$, so the second $\mathcal{N}=8$ supersymmetric action is given by

$$S_{sc}^{(\mathcal{N}=8)} = \frac{1}{2} \left(\int \mu_A^{(-2)} \mathcal{V} \Psi^{+A} \Psi_A^+ - \frac{1}{3} \int dt d^4 \theta v^3 \right). \quad (4.10)$$

Since the first term in (4.10) is $D(2, 1; \alpha)$ invariant at any α , while the second one is invariant under $D(2, 1; \alpha = -1/3)$ (see eq. (3.14)), we conclude that the full action (4.10) is invariant under the $\mathcal{N}=4$ superconformal group $D(2, 1; \alpha = -1/3)$. Since it is also invariant under the rigid $\mathcal{N}=8, d=1$ supersymmetry, it is invariant under some $\mathcal{N}=8$ superconformal group. Therefore the action (4.10), provided that \mathcal{V} and v start with a constant, $\mathcal{V} = 1 + \tilde{\mathcal{V}}$, $v = 1 + \tilde{v}$, defines a new model of $\mathcal{N}=8$ superconformal mechanics associated with the $\mathcal{N}=8$ multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$. It is easy to recognize which $\mathcal{N}=8, d=1$ superconformal group we are facing in the present case. As follows from [16], the only such supergroup in which one can embed $D(2, 1; \alpha = -1/3)$ (to be more exact, an equivalent supergroup $D(2, 1; \beta)$, $\beta = -\frac{1+\alpha}{\alpha} = 2$) is the exceptional $\mathcal{N}=8, d=1$ superconformal group $F(4)$, with the R -symmetry subgroup $SO(7)$.

As already mentioned, the actions (3.25) and (3.26) are the unique $d=1$ sigma model type actions simultaneously involving both the $(\mathbf{0}, \mathbf{4}, \mathbf{4})$ and $(\mathbf{1}, \mathbf{4}, \mathbf{3})$ multiplets. Hence the sum of their $\mathcal{N}=8$ completions, (4.6) and (4.10), yields the most general $\mathcal{N}=8$ supersymmetric sigma-model type off-shell action of the multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ in the $\mathcal{N}=4$ superfield formulation:

$$S_{gen}^{(\mathcal{N}=8)} = \gamma S_{free}^{(\mathcal{N}=8)} + \gamma' S_{sc}^{(\mathcal{N}=8)}, \quad (4.11)$$

γ and γ' being two independent renormalization constants. Surprisingly, it is also $F(4)$ invariant, though with respect to modified superfield transformation laws. Choosing, for simplicity, $\gamma = 1$, and making the redefinitions $1 + \gamma' \mathcal{V} = \tilde{\mathcal{V}}$, $1 + \gamma' v = \tilde{v}$, $\gamma' \Psi^{+A} = \tilde{\Psi}^{+A}$, we observe that (4.11) is reduced, up to a constant renormalization factor, to (4.10) where all superfields are replaced by those with “tilde”. Assuming for the new superfields the same transformation laws (3.17), (3.22) as for the original ones, we see that (4.11) is also $F(4)$ invariant. The $F(4)$ transformations of the original superfields v and \mathcal{V} are of course modified, e.g. $\delta'_{sc} v = -\Lambda_0 [v + (\gamma')^{-1}]$. The transformations of the hidden $\mathcal{N}=4$ supersymmetry remain unaltered. There is no way to make superconformal the free action (4.6), while (4.11) is $\mathcal{N}=8$ superconformal at any $\gamma' \neq 0$.

One can also check that each of the two FI terms, (3.20) and (3.27), is invariant under $\mathcal{N}=8$ supersymmetry (4.2), (4.3). However, they both break superconformal symmetry and $SO(7)$. The same symmetry properties are exhibited by the on-shell potential terms arising upon eliminating the auxiliary fields A^{ik} , a^m and \bar{a}^m from the sum of (4.11) and (3.20), (3.27).

5 Component actions

Using the explicit component expansions (3.4) and (3.21), it is straightforward to find the component form of the superfield actions (3.13) and (3.14)

$$S_{free}^{(\mathcal{N}=8)} = \int dt \left[\dot{x}\dot{x} - \frac{i}{4} (\omega^i \dot{\bar{\omega}}_i - \bar{\omega}^i \dot{\omega}_i) + i\psi^{iA} \dot{\psi}_{iA} + \frac{1}{2} A^{ik} A_{ik} + a^A \bar{a}_A \right] \equiv \int dt \mathcal{L}_{free}^{(\mathcal{N}=8)}, \quad (5.12)$$

$$S_{sc}^{(\mathcal{N}=8)} = \int dt \left[x \mathcal{L}_{free}^{(\mathcal{N}=8)} + \frac{i}{2} A^{ik} \left(\psi_i^A \psi_{kA} + \frac{1}{2} \omega_i \bar{\omega}_k \right) - \frac{1}{2} \left(\bar{\omega}^k \psi_{kA} a^A + \omega^k \psi_k^A \bar{a}_A \right) \right]. \quad (5.13)$$

To interpret (5.13) as the superconformal action, one needs to make the shift $x = 1 + \tilde{x}$. We observe that the general off-shell action (4.11) is specified by the linear function $f(x) = \gamma + \gamma' x$, and its derivative $f_x = \gamma'$, in agreement with the result of [20] where the component off-shell action of the multiplet **(1, 8, 7)** was constructed by a different method. Here we reproduced this result in a manifestly $\mathcal{N}=4$ supersymmetric off-shell superfield formalism. We took advantage of the latter to show the $F(4)$ superconformal invariance of the action (5.13), as well as of a sum of (5.12) and (5.13) (with respect to modified $F(4)$ transformations). Also, we showed that the FI terms (3.20), (3.27) preserve $\mathcal{N}=8$ supersymmetry (although they break $F(4)$). The component expressions for (3.20), (3.27) can be easily found

$$S_{FI}^{(v)} = - \int dt c^{ik} A_{ik}, \quad S_{FI}^{(\psi)} = \int dt (\xi_A \bar{a}^A - \bar{\xi}^A a_A). \quad (5.14)$$

After eliminating the auxiliary fields A^{ik}, a^A, \bar{a}_A in the sum $S_{gen}^{(\mathcal{N}=8)} + S_{FI}^{(v)} + S_{FI}^{(\psi)}$ by their algebraic equations of motion, there appears a scalar potential $\sim (\gamma + \gamma' x)^{-1}$ (plus some accompanying fermionic terms) which is not conformal. Perhaps, conformal potentials could be generated by coupling the multiplet **(1, 8, 7)** to some additional $\mathcal{N}=8$ multiplets.

For completeness, we present the component form of the transformations (4.1)

$$\begin{aligned} \delta_\eta x &= -\eta_{iA} \psi^{iA}, & \delta_\eta \omega^i &= -2\eta^{iA} a_A, & \delta_\eta \bar{\omega}^i &= -2\eta^{iA} \bar{a}_A, & \delta_\eta A^{ik} &= 2\eta^{(iA} \psi_A^{k)}, \\ \delta_\eta \psi^{iA} &= i\eta_k^A A^{ki} - i\eta^{iA} \dot{x}, & \delta_\eta a^A &= i\eta^{iA} \dot{\omega}_i, & \delta_\eta \bar{a}^A &= i\eta^{iA} \dot{\bar{\omega}}_i. \end{aligned} \quad (5.15)$$

It is straightforward to check the invariance of (5.12) and (5.13) under these transformations. Let us also give the R -symmetry transformations belonging to the coset $SO(7)/[SU(2)]^3$, where $[SU(2)]^3$ is the product of three $SU(2)$ symmetries: $SU(2)_{PG}$ acting on the indices A and commuting with the manifest $\mathcal{N}=4$ supersymmetry, manifest R -symmetry $SU(2)_R$ acting on the doublet indices i, k, \dots , and one more hidden $SU(2)_{R'}$ which rotates θ_i through $\bar{\theta}_i$, D^i through \bar{D}^i , ω^i through $\bar{\omega}^i$ and a^A through \bar{a}^A . These transformations read

$$\begin{aligned} \delta_\lambda A^{ik} &= \lambda^{(ik)B} a_B - \bar{\lambda}^{(ik)}_{\bar{B}} \bar{a}^{\bar{B}}, & \delta_\lambda a^B &= -\bar{\lambda}^{(ik)B} A_{ik}, & \delta_\lambda \bar{a}^B &= \lambda^{(ik)B} A_{ik} \\ \delta_\lambda \psi^{iA} &= -\frac{i}{2} \left[\lambda^{(ik)A} \omega_k + \bar{\lambda}^{(ik)A} \bar{\omega}_k \right], & \delta_\lambda \omega^i &= -2i \bar{\lambda}^{(ik)A} \psi_{kA}, & \delta_\lambda \bar{\omega}^i &= 2i \lambda^{(ik)A} \psi_{kA}. \end{aligned} \quad (5.16)$$

Here $\lambda^{(ik)B}, \bar{\lambda}_{(ik)B}$ are 6 complex parameters, which, together with 9 real parameters of $[SU(2)]^3$, are the 21 real parameters of the group $SO(7)$. The bosonic field x is an $SO(7)$ singlet.

6 Conclusions

In this article, using the manifestly $\mathcal{N}=4$ supersymmetric language of the $\mathcal{N}=4, d=1$ harmonic superspace, we constructed a new $\mathcal{N}=8$ superconformal model associated with the off-shell multiplet $(\mathbf{1}, \mathbf{8}, \mathbf{7})$ and showed that the corresponding $\mathcal{N}=8$ superconformal group is the exceptional supergroup $F(4)$. We also found that the generic sigma-model type off-shell action of this multiplet is given by a sum of the superconformal action which is trilinear in the involved $\mathcal{N}=4$ superfields and the free bilinear action. The generic action is also superconformal, but with respect to modified $F(4)$ transformations. The component action is in agreement with the one derived in [20]. The $\mathcal{N}=8$ supersymmetric potential terms can be generated by two superfield FI terms which break both superconformal and $SO(7)$ symmetries. An interesting problem for further study is to see whether superconformal potentials could be generated by coupling this model to some other known $\mathcal{N}=8$ multiplets. Also it would be of interest to find out possible implications of this new superconformal model in the brane, black holes and AdS_2/CFT_1 domains, e.g. along the lines of refs. [4]-[6], [10]-[13].

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